

Closed book. No calculators are to be used for this quiz.  
Quiz duration: 10 minutes

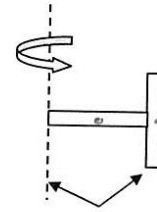
Name:

Student ID:

Signature:

Two identical rods, each of mass  $m$  and length  $L$  are connected such that one is attached to the midpoint of the other in perpendicular direction. Calculate the moment of inertia of this rigid body system around a rotation axis that is passing through the end point of one rod and being parallel to the other rod as shown by the dashed line in figure.

(Hint:  $I_{cm} = mL^2/12$  for a rod of mass  $m$  and length  $L$  for a rotation axis perpendicular to the axis of the rod and passing through its center of mass)



The rotation axis and this rod are parallel

$$I = I_{cm} + m \left(\frac{L}{2}\right)^2 + mL^2$$

$$I = \frac{mL^2}{12} + \frac{mL^2}{4} + mL^2 = \frac{16mL^2}{12} = \frac{4mL^2}{3}$$

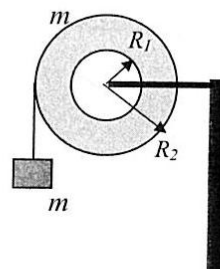
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A block of mass  $m$  is connected to a rope that is wrapped around the rim of a hollow cylinder of mass  $m$ , inner radius  $R_1$  and outer radius  $R_2$ . The cylinder is free to rotate around an axle through the center. At  $t = 0$ , the block is released from rest and moves down under gravitational force. Assume that the rope moves without slipping around the cylinder. What must be the ratio  $R_1/R_2$  if the kinetic energy of the hollow cylinder is %25 of the total kinetic energy during the motion?



(Hint:  $I_{\text{hollow cylinder}} = \frac{M(R_1^2 + R_2^2)}{2}$ )

$$\%25 = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} I \omega^2 + \frac{1}{2} m v^2}$$

$$\frac{1}{4} = \frac{\frac{1}{2} m \left( \frac{R_1^2 + R_2^2}{2} \right) \omega^2}{\frac{1}{2} m \left( \frac{R_1^2 + R_2^2}{2} \right) \omega^2 + \frac{1}{2} m \omega^2 R_2^2}$$

$$\left( \frac{R_1^2}{2} + \frac{3R_2^2}{2} \right) = 2(R_1^2 + R_2^2)$$

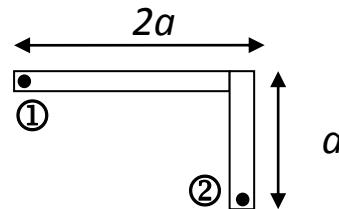
$$R_1^2 + 3R_2^2 = 4R_1^2 + 4R_2^2$$

$$\frac{R_1^2}{R_2^2} = -\frac{1}{3}$$

This negative result implies that it is impossible to have this kinetic energy ratio for the hollow cylinder (or, it is impossible to construct such a hollow cylinder with this mass and radii to have 25% of the kinetic energy in this system).

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Consider two rods of equal mass  $m$ , and of lengths  $2a$  and  $a$ . The rods are joined to form an L-shaped object (assume that the thickness of the rods is negligible). Calculate the ratio  $I_1 / I_2$  where  $I_1, I_2$  are the moment of inertia of the L-shaped object around an axis perpendicular to the plane of L shape (out of the page) and passing through the end points 1 and 2, respectively, as shown in the figure. (Hint:  $I_{cm} = mL^2/12$  for a rod of mass  $m$  and length  $L$ )



Solution:

The L-shaped object can be considered as the combination of two thin rods. So the total moment of inertia will be the sum of moment of inertia of individual rods. Use parallel axis theorem for the axes passing through points 1 and 2:  $I_{axis1, axis2} = I_{cm} + md^2$  where  $I_{cm}$  is the moment of the inertia of a rigid body about an axis passing through its center of mass, and  $d$  is the distance of the axis1 or axis2 to the center-of-mass axis for each rod segment. These axes must be parallel, which is the case in this problem.

$$\text{about axis 1: } I_1 = \underbrace{\frac{m(2a)^2}{12} + ma^2}_{\text{I of rod of length } 2a} + \underbrace{\frac{m(a)^2}{12} + m((2a)^2 + m(\frac{a}{2})^2)}_{\text{I of rod of length } a} = \frac{17}{3}ma^2$$

$$\text{about axis 2: } I_2 = \underbrace{\frac{m(2a)^2}{12} + m(2a^2)}_{\text{I of rod of length } 2a} + \underbrace{\frac{m(a)^2}{12} + m(\frac{a}{2})^2}_{\text{I of rod of length } a} = \frac{8}{3}ma^2$$

hence  $I_1/I_2 = 17/8$

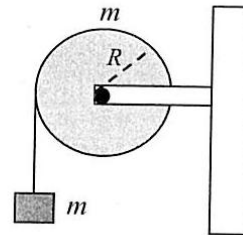
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A block of mass  $m$  is connected to a rope that is wrapped around the rim of a disc of mass  $m$  and radius  $R$ . The disc is free to rotate around an axle through its center. The block is released from rest and descends down under gravitational force. When the block has moved down by a distance  $h$ , calculate the radial acceleration of a point at a distance  $R/2$  from the center of the disc. Assume that the rope moves without slipping around the cylinder. Use the conservation of energy in your solution. ( $I_{\text{disc}} = mR^2/2$ )



$$mgh = \frac{1}{2} \underbrace{I_{\text{disk}}}_{\frac{1}{2}mR^2} \omega^2 + \frac{1}{2} m v^2$$

$\downarrow \frac{\omega^2}{R^2}$

$$gh = \frac{3}{4} \omega^2 R^2 \Rightarrow \omega^2 = \frac{4gh}{3R^2}$$

$a_{\text{rad}}$  at  $R/2$

$$a_{\text{rad}} = \omega^2 \cdot \frac{R}{2} = \frac{4gh}{3R^2} \cdot \frac{R}{2} = \frac{2gh}{3R}$$

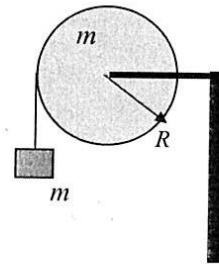
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A block of mass  $m$  is connected to a rope that is wrapped around the rim of a solid cylinder of mass  $m$  and radius  $R$ . The cylinder is free to rotate about an axle passing through its center. At  $t = 0$ , the block is released from rest and moves down under gravitational force. Assume that the rope moves without slipping around the cylinder. Calculate the percent of the total kinetic energy that the



block has during the motion. ( $I_{\text{solid cylinder}} = \frac{MR^2}{2}$ )

$$K = \underbrace{\frac{1}{2} I \omega^2} + \underbrace{\frac{1}{2} m v^2}_{\text{block KE during motion}}$$

$$\% = \frac{\frac{1}{2} m (\omega R)^2}{\frac{1}{2} \frac{m R^2}{2} \omega^2 + \frac{1}{2} m (\omega R)^2} \times 100$$

$$= \frac{\frac{1}{2} m \omega^2 R^2}{\frac{3}{4} m \omega^2 R^2} \times 100 \approx 66 \%$$